# Exam in <br> SSY305 Communication Systems 

Department of Electrical Engineering

Exam date: June 9, 2023

## Teaching Staff

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Material Allowed material:

- Chalmers-approved calculator.
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photocopies, printouts, other students' notes, or any other material is not allowed.
- A paper-based dictionary without added notes (electronic dictionaries are not allowed).

Students are required to solve the exam problems individually. Cooperation, in any form or with anyone, is strictly forbidden.

Grading A correct, precise, and well-motivated solution gives at most 12 points per problem. An incorrect answer, unclear, incomplete, or poorly motivated solutions give point reductions to a minimum of 0 points. No fractional points are awarded. Answers in any other language than Swedish or English are ignored.

Solutions Are made available at the earliest on June 12 EOB on the course web page.
Results Exam results are posted on Canvas no later than June 23. The grading review process will be explained on the course webpage.

Grades The final grade of the course will be decided by the project (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). The project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade.

| Total Score | $0-39$ | $40-68$ | $69-79$ | $\geq 80$ |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Fail | 3 | 4 | 5 |

## Table over the Q-function

|  | Q(x) |  | Q(x) |  | Q(x) |  | $Q(x)$ |  | Q(x) |  | Q(x) |  | Q(x) |  | $Q(x)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | $5.0000 \mathrm{E}-01$ | 0.76 | $2.2360 \mathrm{E}-01$ | 1.52 | $6.4260 \mathrm{E}-02$ | 2.28 | $1.1300 \mathrm{E}-02$ | 3.04 | $1.1830 \mathrm{E}-03$ | 3.80 | 7.2350E-05 | 4.56 | $2.5580 \mathrm{E}-06$ | 5.32 | 5.1880 E |
| 0.01 | $4.9600 \mathrm{E}-01$ | 0.77 | $2.2060 \mathrm{E}-0$ | 1.5 | $6.3010 \mathrm{E}-02$ | 2.29 | 1.1 | 3.05 | $1.1440 \mathrm{E}-03$ | 3.81 | $6.9480 \mathrm{E}-05$ | 4.57 | $2.4390 \mathrm{E}-06$ | 3 |  |
| 0.02 | 4.9200 | 0. | $2.1770 \mathrm{E}-01$ |  | $6.1780 \mathrm{E}-02$ | 2. | $1.0720 \mathrm{E}-02$ | 3.0 | $1.1070 \mathrm{E}-03$ | 3.8 | 5 | 8 | 6 | 5.34 | $4.6470 \mathrm{E}-08$ |
| 0.03 | $4.8800 \mathrm{E}-01$ | 0.79 | $2.1480 \mathrm{E}-01$ | 1.55 | 6.0 | 2.3 | 1.0 | 3.07 | $1.0700 \mathrm{E}-03$ |  | 5 | 4.59 | $2.2160 \mathrm{E}-06$ | 5.35 | $4.3980 \mathrm{E}-08$ |
| 0.04 | $4.8400 \mathrm{E}-01$ | 0.8 | 2.1 | 1.5 | 5.938 | 2.3 | 1.01 | 3.08 | 1.03 | 3.8 | 6.1 | 4.60 | $2.1120 \mathrm{E}-06$ | 5.36 | $4.1610 \mathrm{E}-08$ |
| 0.05 | 4.8010 | 0.81 | 2.0900 | 1.57 | 5.82 | 2.33 | 9.9030 | 3.09 | 1.00 | 3.85 | 5.9060 | 4.61 | 30 | 5.3 | $3.9370 \mathrm{E}-08$ |
| . 06 | 4.761 | 0.82 | 2.061 | 1.58 | 5.7 | 2.34 | 9.6 | 3.10 | 9.6760E-04 | 3.86 | $5.6690 \mathrm{E}-05$ | 4.6 | 6 | 8 | $3.7240 \mathrm{E}-08$ |
| 0.07 | 4.72 | 0.83 | 2.0330E-01 | 1.59 | 5. | 2. | 9. | 3.11 | $9.3540 \mathrm{E}-04$ | 3.87 | 5.4 | 4.63 | 1.8280E-06 | 5.39 | $3.5230 \mathrm{E}-08$ |
| 0.08 | 4.6 | 0.84 | 2.0 | 1.60 | 5. | 2.3 | 9.1 | 3.12 | $9.0430 \mathrm{E}-04$ | 3.8 | $5.2230 \mathrm{E}-05$ | 4.64 | 1.7420E-06 | 5.40 | 3.3 |
| 0.09 | $4.6410 \mathrm{E}-01$ | 0.85 | 770 | 1.61 | $5.3700 \mathrm{E}-02$ | . 3 | 8.8940 | 3.1 | 7400E-04 | 3.89 | $5.0120 \mathrm{E}-05$ | 4.6 | 6600E-06 | 5.4 | 3.1510E-08 |
| 0.10 | 4.6020 | 0.86 | 1.9490E-01 | 1.62 | 5. | 2.3 | 8.6 | 3.14 |  | 3.90 | 5 | 6 | 6 | 5.42 | $2.9800 \mathrm{E}-08$ |
| 0.11 | 4.5620 | 0.87 | $1.9220 \mathrm{E}-01$ | 1.63 | 5. | 2.39 | 8. | 3.15 | 8.1640E-04 | 3.9 | $4.6150 \mathrm{E}-05$ | 4.67 | 1.5060E-06 | 5.43 | $2.8180 \mathrm{E}-08$ |
| 0.12 | 4. | 0.88 | 1.8940E-0 | 1.64 | 5. | 2.40 | 8. | 3.16 | 7.888 | 3.9 | 4. | 4.68 | $1.4340 \mathrm{E}-06$ | 5.44 | 2.6 |
| 0.13 | $4.4830 \mathrm{E}-01$ | 0.8 | 1.8670E-01 | 1.65 | 4.9470 | 2.41 | 7.9760 | 3.17 | 6220 | 3.93 | $4.2470 \mathrm{E}-05$ | 4.69 | 3660E-06 | 5.45 | $2.5180 \mathrm{E}-08$ |
|  | 4.44 | 0 |  | 1. | 4.8460E-02 | 2.42 | $7.7600 \mathrm{E}-03$ | 3. | $7.3640 \mathrm{E}-04$ | 3.94 | $4.0740 \mathrm{E}-05$ | 0 | 1.3010E-06 |  | $2.3810 \mathrm{E}-08$ |
| 0.15 | 4.40 | 0 | $1.8140 \mathrm{E}-01$ | 1.67 | 4. | 2.43 | 7. | 3. | $7.1140 \mathrm{E}-04$ | 3.9 | 3.9 | 4.71 | 1.2390E-06 | 5.47 | $2.2500 \mathrm{E}-08$ |
| 0.16 | 4.364 | 0.92 | $1.7880 \mathrm{E}-01$ | 1.68 | 4. | 2.44 | 7. | 3. | 6. | 3.96 | 3.7 | 4.72 | $1.1790 \mathrm{E}-06$ | 5.48 | $2.1270 \mathrm{E}-08$ |
| 0.17 | 4.3250 | 0.93 | 1.7620E-01 | 1.69 | $4.5510 \mathrm{E}-02$ | 2.4 | 7.1430E-03 | 3.21 | 63 | 3.97 | $3.5940 \mathrm{E}-05$ | 4.73 | 1230E-06 | 9 | $2.0100 \mathrm{E}-08$ |
| 0.18 | 4.286 | 0 | 1.7360E-01 |  | $4.4570 \mathrm{E}-02$ | 2.46 |  | 3. |  | 3.9 | $3.4460 \mathrm{E}-05$ | 4.74 | 6 | 5.50 |  |
| 0.19 | 4.247 | 0.95 | 1.7110E-01 | 1.71 | 4.363 | 2.47 | 6. | 3.23 | $6.1900 \mathrm{E}-04$ | 3.9 | 3.3 | 4.75 | $1.0170 \mathrm{E}-06$ | 5.51 | 1.7 |
| 0.20 | 4.2070 | 0. | 1.6850 | 1. | 4.2720 | 2.48 | 6.5690 | 3.24 | 5.9760 | 4.00 | $3.1670 \mathrm{E}-05$ | 4.76 | 9.6800 | 5.52 | 1.6950E-08 |
| 0.21 | 4.168 | 0 | 1.6600 | 1. | 4.1820E-02 | 2.4 | $6.3870 \mathrm{E}-03$ | 3.25 |  | 4.01 | 3.0 | 7 | $9.2110 \mathrm{E}-07$ | 5.53 | $1.6010 \mathrm{E}-08$ |
| 0.22 | 4.1290 | 0.9 | 1.6350E-01 | 1.7 | 4.0930E-02 | 2.50 |  | 3.2 |  | 4.02 | 2.9 | 4.78 | $8.7650 \mathrm{E}-07$ |  | $1.5120 \mathrm{E}-08$ |
| 0.23 | 4.0900 | 0.99 | 1.6110 | 1.75 | 4.0060E-02 | 2.51 | 6.0 | 3. | 5.3 | 4.03 | 2.78 | 4.79 | 8.33 | 5.55 | 1.4 |
| 0.24 | 4.0520 | 1.00 | 1.5 | 1.76 | 3.9200 | 2.52 | 5.868 | 3.2 | 5.19 | 4.04 | $2.6730 \mathrm{E}-05$ | 4.80 | 9330E | 5.56 | .3490E-08 |
| 0.25 | 4.0130 | 1.0 | 1.5620 | 1. | . 83 | 2.5 | 5.7030 | 3. | 5.0090E-04 | 5 | 2.5 | 4.8 | $7.5470 \mathrm{E}-07$ | 5.57 | $1.2740 \mathrm{E}-08$ |
| 0.26 | 3.97 |  | 1.5390E-01 | 1.7 | $3.7540 \mathrm{E}-02$ |  |  | 3. | $4.8340 \mathrm{E}-04$ | 4.06 | 2. | 4.82 | $7.1780 \mathrm{E}-07$ | 5.58 | $1.2030 \mathrm{E}-08$ |
| 0.27 | 3.9360 | 1.03 | 1.5150 | 1.79 | 3. | 2.5 | 5.3 | 3.3 | 665 | 4.07 | 2.3510 |  | 6.8270 | 5.59 | $1.1350 \mathrm{E}-08$ |
| 0.28 | 3.897 | 1.04 | 20 | 1.80 | 3. | 2.56 | $5.2340 \mathrm{E}-03$ | 3.32 | 501 | 4.08 | $2.2520 \mathrm{E}-05$ | 4.84 | $6.4920 \mathrm{E}-07$ | . 6 | 08 |
| 0.29 | 3.859 | 1.05 | 1.4690 | 1.8 | 3.5150 | 2.5 | 5.0850 | 3.33 | $4.3420 \mathrm{E}-04$ | 4.09 | 2.1 | 4.8 | $6.1730 \mathrm{E}-07$ | 5.61 | $1.0120 \mathrm{E}-08$ |
| 0.30 | 3.8 |  | $1.4460 \mathrm{E}-01$ | 1.8 | $3.4380 \mathrm{E}-02$ | 2.5 | $4.9400 \mathrm{E}-03$ | 3.34 | $4.1890 \mathrm{E}-04$ | 4.10 | 2.0 |  | $5.8690 \mathrm{E}-07$ | 5.62 | 9.5 |
| 0.31 | 3.783 | 1.07 | $1.4230 \mathrm{E}-01$ | 1.83 | 3.3620 | 2.59 | $4.7990 \mathrm{E}-03$ | 3. | 4. | 4.11 | $1.9780 \mathrm{E}-05$ |  | 5.5800 | 5.63 | 9.0100E-09 |
| 0.32 | 3.745 | 1. | 1.4010 | 1.8 | 3.2880E-02 | 2.6 | $4.6610 \mathrm{E}-03$ | 3.36 | 3.8970E-04 | 4.12 | $1.8940 \mathrm{E}-05$ | 4.88 | $5.3040 \mathrm{E}-07$ | 5.64 | .5030E-09 |
| 0.33 | 3.70 | 1. | 1.3790 | 1.8 | 3.2160 | 2.6 | 4.52 | 3.3 | $3.7580 \mathrm{E}-04$ | 4.13 | $1.8140 \mathrm{E}-05$ | 4.89 | 5.0420E-07 | 5.65 | $8.0220 \mathrm{E}-09$ |
|  | 3.6 |  |  |  |  | 2.62 |  | 3.38 |  |  |  | 4.90 | 7 |  |  |
| 0.35 | $3.6320 \mathrm{E}-01$ |  | 35 | 1.87 | 3. | 2.63 | $4.2690 \mathrm{E}-03$ | 3.39 | 3.4950 | 4.15 | 1.6620 | 4. | 5540E-07 | 5.67 | .1400E-09 |
| 0.36 | 3.5940 | 1. | 1.31 | 1. | . 005 | 2.6 | 4.1450 | 3.40 | 369 | 4.16 | 1.59 | 4.9 | 327 | 5.68 | .7350E-09 |
| 0.37 | 3.5570 |  | 1.2920 |  | 2.938 |  | 4.0250 |  | $3.2480 \mathrm{E}-04$ | 4.17 | 1.52 | 4. | $4.1110 \mathrm{E}-07$ | 5.69 | $6.3520 \mathrm{E}-09$ |
|  | 3.52 |  |  |  | 2. | 2.66 |  |  | 3.1 | - | 1.4 |  | $3.9060 \mathrm{E}-07$ | 5.7 | 5.9900E-09 |
| 0.39 | 3.4830 | 1.1 | $1.2510 \mathrm{E}-01$ | 1.91 | 2.8070 | 2.67 | $3.7930 \mathrm{E}-03$ | 3.43 | 3.0180E-04 | - | $1.3950 \mathrm{E}-05$ | 4.95 | $3.7110 \mathrm{E}-07$ | 5.71 | 09 |
| 0.40 | 3.4460 | 1.1 | 1.230 | 1.92 | $2.7430 \mathrm{E}-02$ | 2.6 | 3.6 |  | 2.90 | - | 1.3350 | 4. | 3.5250 | 5.72 | 5.32 |
|  | 3.4090 |  | 1.2100E-01 | 1.93 | $2.6800 \mathrm{E}-02$ | 2.6 |  |  | 2.8 |  | 1.2 |  | , |  | $5.0220 \mathrm{E}-09$ |
|  | 3.3 | 1.18 | $1.1900 \mathrm{E}-01$ |  | 2. | 2.70 | 3. | 3.46 | $2.7010 \mathrm{E}-04$ | 4.22 | 20E-05 | 4.98 | $3.1790 \mathrm{E}-07$ | 5.74 | 09 |
|  | 3.3360 | 1.19 | 1.1700 | 1.9 | 2.5590 E | 2.7 | 3.3640 | 3.47 | 2.6 | 4.23 | $1.1680 \mathrm{E}-05$ | - | 3.0190 | 5.75 | 09 |
|  | 3.3000 | 1. | 1.1510 | 1.9 | 2.500 | 2.72 | 3.2 | 3. | 2.50 | 4.24 | $1.1180 \mathrm{E}-05$ | 5.00 | 2.8670 | 5.76 | . 2 |
|  | 3.2 |  |  |  |  | 2.7 |  |  | 2. |  | $1.0690 \mathrm{E}-05$ |  | , | 5.77 | $3.9640 \mathrm{E}-09$ |
|  | 3.2 |  | 1.1120 | 1.98 | 2.3850 E |  | $3.0720 \mathrm{E}-03$ | 3.50 | 2. |  | 5 | 5.02 | $2.5840 \mathrm{E}-07$ | 5.78 | 09 |
|  | 3.1920 | 1.23 | 93 | 1.99 | . 33 | 2.7 | 2.98 | 3.51 | 2.241 | 4.27 | $9.7740 \mathrm{E}-06$ | 5.03 | 2.4520 | 5.79 | 3.5 |
|  | 3.1560 |  | 1.0750 E | 2.0 | 2.2750 E | 2.7 | 2.8 | 3.52 | 2.15 | 4.28 | $9.3450 \mathrm{E}-06$ | 5. | $2.3280 \mathrm{E}-07$ | 5.80 | 3. |
|  | 3.1210 | 1.25 | $1.0560 \mathrm{E}-01$ | 2. | 2. | 2. | 2.8 |  | 2.0 |  | 8. | 5. | $2.2090 \mathrm{E}-07$ | 5.81 | $3.1240 \mathrm{E}-09$ |
|  | 3.0850 | 1.26 | 1.0380E-01 | 2.02 | 2.1690 | 2.7 | $2.7180 \mathrm{E}-03$ |  | . 001 | 4.30 | $8.5400 \mathrm{E}-06$ | 5.06 | - | 5.82 | -09 |
|  | 3.0500 | 1. | 1.0200 | 2.03 | . 1 | 2.7 | 2.6350 | 3. | 1.9260E-04 | 4.31 | 8.16 | 5.07 | 9890 | 5.83 | 2.7 |
|  | 3.01 | 1.2 | 1.0030E-01 | 2.0 | 2. | 2.8 | 2. | 3.56 | $1.8540 \mathrm{E}-04$ | 4.32 | $7.8010 \mathrm{E}-06$ | 8 | 1.8870E-07 |  | $2.6100 \mathrm{E}-09$ |
| . | 2.9810 | 1.29 | 9.8530 | . 0 | 2.0180 | 2.8 | $2.4770 \mathrm{E}-03$ | 3.5 | 50 | 4.33 | 4550 | 5.0 | 7900E-07 | 5.8 | $2.4580 \mathrm{E}-09$ |
|  | 2.9460 | 1.30 | $9.6800 \mathrm{E}-02$ | 2.0 | 1.9700 | 2.8 | . 40 | 3.5 | 1.7180E-04 | 4.3 | 7.1240 | 5. | 6980 | 5.86 | $2.3140 \mathrm{E}-09$ |
|  | 2.91 | 1.3 | 9.5100 | 2.07 | 1.9 | 2.83 | 2.32 | 3.5 | $1.6530 \mathrm{E}-04$ | 4. | 6.807 | 5. | 6110 | 5.87 | 2.1 |
|  | 2.8 | 1.32 | 9.3 | 2. | 1. | 2.8 | 2. | 3.6 | 1.5 |  | 6.5 | 5. | 1.5280E-07 | 5.88 | $2.0510 \mathrm{E}-09$ |
| 0.57 | 2.8430E-01 | 1. | 9.1760 | 2.09 | 1.831 | 2.85 | . 1860 | 3.6 | 1.5310 E | 4.37 | $6.2120 \mathrm{E}-06$ | 5. | $4490 \mathrm{E}-07$ | 5.8 | 1.9310E-09 |
| 0.58 | 2.8100 | 1. | 9.0120E-02 | 2.10 | 1.78 | 2.86 | 2.1180 | 3.6 | 1.47 | 4.3 | 5.9340 | 5. | 1.3740 | 5.90 | 1.8180E-09 |
|  | 2.7 | 1.35 | 8.8510E-02 | 2.1 | 1.7430 | 2.8 | 2.0520 | 3. | $1.4170 \mathrm{E}-04$ | 4.39 | 5.6 | 5. | 3020 | 5.91 | 1.7 |
| 0.60 | $2.7430 \mathrm{E}-01$ | 1. | 8.6910 | 2.1 | 1.7000 | 2.8 | 1.988 | 3.6 | 1.3630 | 4. | 5.4130 | 5.16 | $1.2350 \mathrm{E}-07$ | 5.9 | 1.6100E-09 |
|  | $2.7090 \mathrm{E}-01$ | 1.3 | 8.5340 | 2.13 | 1.6590 | 2.89 | 260 | 3.65 | 1.3110 | 4.4 | 5.1690 | 5.1 | $1.1700 \mathrm{E}-07$ | 5.93 | 1.5150E-09 |
| 0.62 | 2.676 | 1.38 | 37 | 2.1 | 1.61 | 2.90 | 1.8660E-03 | 3.6 | 1.26 | 4.4 | 4.9350 | 5.1 | 1090 | 5.94 | $1.4250 \mathrm{E}-09$ |
| 0.63 | 2.6 | 1.39 | 8. | 2.15 | 1.5 | 2.91 | 1. | 3.6 | 1.2 | 4.43 | $4.7120 \mathrm{E}-06$ | 5. |  | 5.95 | $1.3410 \mathrm{E}-09$ |
| 0.64 | $2.6110 \mathrm{E}-01$ | 1.40 | 8.0760 | 2.16 | 1.5390E | 2.9 | 1.7500 | 3.68 | $1.1660 \mathrm{E}-0$ | 4.4 | $4.4980 \mathrm{E}-06$ | 5.20 | $9.9640 \mathrm{E}-08$ | 5.96 | $1.2610 \mathrm{E}-09$ |
| 0.65 | $2.5780 \mathrm{E}-01$ | 1.4 | 9270 | 2.1 | 1.5000 | 2.9 | .6950 | 3.6 | 1.1210 | 4.4 | .2940 | 5.2 | $9.4420 \mathrm{E}-08$ | 5.9 | $1.1860 \mathrm{E}-09$ |
| 0.66 | 2.5460 |  | 800 | . 18 | 1.4630 | 2.94 | 641 | 3.70 | 1.0780 | 4.46 | 4.0980 | 5.2 | $8.9460 \mathrm{E}-08$ | 5.9 | $1.1160 \mathrm{E}-09$ |
| 0.67 | 2.5140 E | 1.43 | 360 | 2.19 | 1. | 2.95 | 1.589 | 3. | 1.0360 | 4. | 3.91 | 5.23 | 8.4760 | 5. | $1.0490 \mathrm{E}-09$ |
| 68 | $2.4830 \mathrm{E}-01$ | 1.44 | 4930 | 2.20 | 1.3900 E | 2.9 | 1.5380 | 3.7 | $9.9610 \mathrm{E}-0$ | 4.48 | $3.7320 \mathrm{E}-06$ | 5.24 | $8.0290 \mathrm{E}-08$ | 6.00 | $9.8660 \mathrm{E}-10$ |
| . 69 | $2.4510 \mathrm{E}-01$ | 1. | 3530E | 2.2 | 1.3550E | 2.9 | 1.4890 | 3.7 | 9.5740 | 4.4 | 3.5610 | 5.2 | 7.6050 E | 6.0 | 9.2760 |
| 0.70 | $2.4200 \mathrm{E}-01$ | 1.46 | 2150 | 2.22 | .3210 | 2.9 | $1.4410 \mathrm{E}-03$ | 3.7 | 9.2010 | 4.50 | 3.3980 | 5.26 | $7.2030 \mathrm{E}-08$ | 6.0 | $8.7210 \mathrm{E}-10$ |
| . 71 | $2.3890 \mathrm{E}-01$ | 1. | 7.0780 | 2.23 | 1.2870E | 2.99 | 1.3950 E | 3.7 | 8.8420E-05 | 4.51 | $3.2410 \mathrm{E}-06$ | 5.27 | $6.8210 \mathrm{E}-08$ | 6.03 | $8.1980 \mathrm{E}-10$ |
| 72 | $2.3580 \mathrm{E}-01$ | 1.48 | $6.9440 \mathrm{E}-0$ | 2.24 | 1.2550E-02 | 3.00 | $1.3500 \mathrm{E}-03$ | 3.76 | 8.4960E-05 | 4.52 | 3.0920E-06 | 5.28 | $6.4590 \mathrm{E}-08$ | 6.04 | 7.7060E-10 |
| 73 | $2.3270 \mathrm{E}-01$ | 1.49 | 6.8110 | 2.25 | $1.2220 \mathrm{E}-02$ | 3.01 | 1.3060 | 3.77 | $8.1620 \mathrm{E}-05$ | 4.53 | $2.9490 \mathrm{E}-06$ | 5.29 | $6.1160 \mathrm{E}-08$ | 6.05 | 7.2420 |
| 0.74 | $2.2960 \mathrm{E}-01$ | 1.50 | 6.6810E-02 | 2.26 | $1.1910 \mathrm{E}-02$ | . 02 | 1.2640 E | 3.78 | $7.8410 \mathrm{E}-05$ | 4.54 | $2.8130 \mathrm{E}-06$ | 5.30 | $5.7900 \mathrm{E}-08$ | 6.06 | $6.8060 \mathrm{E}-10$ |
| 0.75 | $2.2660 \mathrm{E}-01$ | 1.51 | 6.5520 E | 2.27 | $1.1600 \mathrm{E}-02$ | 3.03 | $1.2230 \mathrm{E}-03$ | 3.79 | $7.5320 \mathrm{E}-05$ | 4.55 | $2.6820 \mathrm{E}-06$ | 5.31 | $5.4810 \mathrm{E}-08$ | 6.07 | $6.3960 \mathrm{E}-1$ |

1. Consider a network consisting of a music server, a Wi-Fi router, and a Wi-Fi speaker, as depicted in the figure below. The music server is connected to the Wi-Fi router with a twisted pair cable. The Wi-Fi router and the Wi-Fi speaker are connected wirelessly.


The protocols used for streaming music to the speaker are (in alphabetic order) 802.11 MAC, 802.11 PHY, 802.3 MAC, 802.3 PHY, IP, LLC, RTP, and UDP.
(a) Copy the figure above and put the protocols in their correct places in the protocols stacks. Note that some boxes in the protocol stacks might be empty. (2p)
(b) Let the IP addresses in the network be of the form 192.168.1.xxx, where xxx is a member of the set $\{0,1, \ldots, 255\}$. Assign IP addresses to the appropriate places in the figure. Use IP addresses in order, i.e., start with assigning 192.168.1.0, then 192.168.1.1, and so on. (2p)
(c) Let the physical addresses in the network be of the form 30:23:03:00:00:yy, where yy is a member of the set $\{00,01, \ldots, F F\}$ (all two-digit hex numbers). Assign physical addresses to the appropriate places in the figure. Use physical addresses in order, i.e., start with assigning 30:23:03:00:00:00, then 30:23:03:00:00:01, and so on. (2p)
(d) Let PH denote the header and PT denote the trailer (if it exists) for protocol P. For example, the IP header is denoted as IPH, and the 802.3 trailer is denoted as802.3T. Moreover, let the payload (music bits) be denoted as MUSIC. Indicate the order of the headers, trailer, and payload on a frame going from the music server to the Wi-Fi router. Do the same for the frame from the Wi-Fi router to the Wi-Fi speaker. (2p)
(e) What destination and source IP addresses are used in the frame from the music server to the Wi-Fi router and the frame from the Wi-Fi router to the Wi-Fi speaker? (2p)
(f) What destination and source physical addresses are used in the frame from the music server to the Wi-Fi router and the frame from the Wi-Fi router to the Wi-Fi speaker? (2p)

Hint: RTP stands for Real-time Transport Protocol. It requires the services of a transport layer protocol.

Solution Problem 1

2. Let's assume to have a systematic block code for error detection when transmitting frames over a noisy channel. A frame is represented by a codeword formed as follows: $\mathbf{c}=\left[\begin{array}{ll}\mathbf{d} & \mathbf{q}\end{array}\right]$ where $\mathbf{d}=\left[\begin{array}{lll}d_{0} & d_{1} & d_{2}\end{array}\right]$ are the information bits and $\mathbf{q}=\left[\begin{array}{ll}q_{0} & q_{1}\end{array}\right]$ are the parity bits. The list of codewords used for transmission is the following:
$\left.\begin{array}{llllll} & & d_{0} d_{1} d_{2} & q_{0} q_{1} \\ \hline c_{1} & 0 & 0 & 0 & & 1 \\ c_{2} & 0 & 0 & 1 & & 1 \\ c_{3} & 0 & 1 & 0 & & 0 \\ c_{4} & 0 & 1 & 1 & & 0 \\ c_{5} & 1 & 0 & 0 & & 0 \\ c_{0} & 1 \\ c_{6} & 1 & 0 & 1 & & 0\end{array}\right)$

The channel introduces independent bit errors with probability $p$.
(a) Is c a linear block code? Motivate. (2p)
(b) Suppose the noisy channel works at rate $R=10$ [Gbit/s]. Compute the value of the information bit rate when $p=0$. ( 2 p )
(c) What is the maximum number of bit errors per frame that this block code is guaranteed to detect? Motivate (4p)
(d) Assume that the transmitted codeword is $c_{5}$. Compute the probability that an undetected frame error occurs when $p=10^{-4}$. (4p)

## Solution Problem 1

(a) According to the definition we discussed in class, a block code $\mathbf{c}$ is linear if

$$
c_{k} \oplus c_{l}=c_{j} \in \mathbf{c}, \forall c_{k} \in \mathbf{c}, \forall c_{j} \in \mathbf{c}
$$

Take now $c_{1} \oplus c_{2}$, as an example. The result is [00101] which is not a codeword. For this reason, $\mathbf{c}$ is not a linear block code.
(b) Out of 5 bits in a frame only 3 are information bits. The information bit rate can be computed as :

$$
R_{i}=\frac{3}{5} \cdot R=0.6 \cdot 10 \cdot 10^{9}=6[\mathrm{Gbit} / \mathrm{s}] .
$$

(c) To compute the minimum number of bit errors that can be detected we need to compute the value of the minimum distance of the code. Since the code is not linear, we have to compute the distance between each codeword pair and then check what the minimum value is.

|  | $c_{i} \oplus c_{j}$ | distance |
| :---: | :---: | :---: |
| $c_{1} \oplus c_{2}$ | 00101 | 2 |
| $c_{1} \oplus c_{3}$ | 01011 | 3 |
| $c_{1} \oplus c_{4}$ | 01110 | 4 |
| $c_{1} \oplus c_{5}$ | 10010 | 2 |
| $c_{1} \oplus c_{6}$ | 10111 | 4 |
| $c_{1} \oplus c_{7}$ | 11001 | 3 |
| $c_{1} \oplus c_{8}$ | 11100 | 3 |
| $c_{2} \oplus c_{3}$ | 01110 | 4 |
| $c_{2} \oplus c_{4}$ | 01011 | 3 |
| $c_{2} \oplus c_{5}$ | 10111 | 4 |
| $c_{2} \oplus c_{6}$ | 10010 | 2 |
| $c_{2} \oplus c_{7}$ | 11100 |  |
| $c_{2} \oplus c_{8}$ | 11001 | 3 |
| $c_{3} \oplus c_{4}$ | 00101 | 2 |
| $c_{3} \oplus c_{5}$ | 11001 | 3 |
| $c_{3} \oplus c_{6}$ | 11100 | 3 |
| $c_{3} \oplus c_{7}$ | 10010 | 2 |
| $c_{3} \oplus c_{8}$ | 10111 | 4 |
| $c_{4} \oplus c_{5}$ | 11100 | 3 |
| $c_{4} \oplus c_{6}$ | 11001 | 3 |
| $c_{4} \oplus c_{7}$ | 10111 | 4 |
| $c_{4} \oplus c_{8}$ | 10010 | 2 |
| $c_{5} \oplus c_{6}$ | 00101 | 2 |
| $c_{5} \oplus c_{7}$ | 01011 |  |
| $c_{5} \oplus c_{8}$ | 01110 | 3 |
| $c_{6} \oplus c_{7}$ | 01110 | 3 |
| $c_{6} \oplus c_{8}$ | 01011 | 3 |
| $c_{7} \oplus c_{8}$ | 00101 | 2 |

The minimum value of the distance is 2 . This means that the code is guaranteed to be able to detect all the single-bit error patterns.
(d) We have an undetected frame error if, after the occurrence of two or more bit errors, we receive a valid codeword. When transmitting [10001] the following undetectable error patterns can happen :

| $c_{i} \oplus c_{j}$ | Undetechable Error patterns | Probability |
| :--- | :--- | :--- |
| $c_{5} \oplus c_{1}$ | 10010 | $p^{2} \cdot(1-p)^{3}$ |
| $c_{5} \oplus c_{2}$ | 10111 | $p^{4} \cdot(1-p)$ |
| $c_{5} \oplus c_{3}$ | 11001 | $p^{3} \cdot(1-p)^{2}$ |
| $c_{5} \oplus c_{4}$ | 11100 | $p^{3} \cdot(1-p)^{2}$ |
| $c_{5} \oplus c_{6}$ | 00101 | $p^{2} \cdot(1-p)^{3}$ |
| $c_{5} \oplus c_{7}$ | 01011 | $p^{3} \cdot(1-p)^{2}$ |
| $c_{5} \oplus c_{8}$ | 01110 | $p^{3} \cdot(1-p)^{2}$ |

The last column expresses the probability a specific error pattern happens, e.g., the 10010 error pattern making $c_{5}=c_{1}$ happens with probability $p^{2} \cdot(1-p)^{3}$. The probability that an undetected frame error occurs is the sum of the probabilities of all these possible error patterns:

$$
p^{4} \cdot(1-p)+4 p^{3} \cdot(1-p)^{2}+2 p^{2} \cdot(1-p)^{3}=2 \cdot 10^{-8}
$$

3. Consider a network where fours LANs (i.e., L1, L2, L3, and L4) are connected via two LAN transparent bridges $(A$ and $B)$. Each LAN comprises two hosts connected and sharing the same medium, as depicted in the figure. All LANs support $100 \mathrm{Mbit} / \mathrm{s}$ transmission. The propagation speed in the medium is $c=2 c_{0} / 3$, where $c_{0}=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$. The bridges operate in a store-and-forward mode. More specifically, an incoming frame is completely received before it is forwarded to the corresponding output port. We assume that the processing time in the bridges and hosts is negligible. The numbers inside each bridge box close to where the transmission cable is attached represent the LAN bridge port number.

(a) The forwarding tables of the LAN bridges are defined as follows:

| Host address | Port ID |
| :---: | :---: |
| $a$ | $x$ |
| $b$ | $y$ |
| $\vdots$ | $\vdots$ |

In other words, whenever a LAN bridge receives a frame for a host with address $a$, it sends it to port $x$, etc. Suppose the LAN bridges are powered up at time $t=0$, which implies that the forwarding tables are empty. Explain in detail how the forwarding tables in bridge $A$ and $B$ change for each of the following transmissions.

| Time $t[\mathbf{s}]$ | Source Host | Destination Host |
| :---: | :---: | :---: |
| 1 | H1 | H7 |
| 2 | H3 | H5 |
| 3 | H4 | H3 |
| 4 | H2 | H4 |
| 5 | H8 | H1 |

Assume that backward learning is used. (4p)
(b) Assume now a different scenario where data traffic is generated between H2 and H3 only. The distance between a host and the bridge port to which it is connected is 50 m . Assume that the medium access control method chosen is CSMA-CD. Compute the minimum frame length $n_{f}$ in bytes that should be used for transmission at the MAC layer at H 2 and H3. Assume now that LAN bridges $A$ and $B$ are replaced by two network hubs (i.e., repeaters). Compute again the minimum frame length $n_{f}$ in byes that should be used for transmission at the MAC layer at H2 and H3. What can you conclude when you compare these latest results? (4p)
(c) Assume that after some time, the network grows in size, and 3 additional bridges $(C, D$, and $E)$ are added to connect 2 new LANs (L5 and L6) as in the figure below.


Using the Spanning Tree algorithm, identify the following (1p):

- the root bridge,
- the root ports in the network,
- for each LAN, the designated bridge and its associated designated port.

While computing the spanning tree, assume the following. Ties should be resolved using the bridge ID or the port ID. More specifically, bridges or ports with the lowest ID value are always chosen first. The cost of a path to the root bridge is equal to the sum of the costs of the traversed LANs. Assume the following cost values:

- Cost of traversing L1 is C1 $=2$
- Cost of traversing L2 is $\mathrm{C} 2=5$
- Cost of traversing L3 is C3 $=1$
- Cost of traversing L4 is $\mathrm{C} 4=3$
- Cost of traversing L5 is C5 $=1$
- Cost of traversing L6 is $\mathrm{C} 6=4$

Draw the spanning tree as a result of the Spanning Tree algorithm just computed. (1p)
(d) If we want, as a result of the spanning tree algorithm, L5 to be connected to the root bridge only, what is the minimum value that C 5 , as defined in part (c), should have, assuming all other costs are constant? Motivates your answer (2p)

## Solution Problem 3

(a) At $t=1$, H1 sends a frame to H7. The frame is received by Bridge A on port 1. The forwarding table of Bridge A (FWD-A) is empty, so H1 is saved together with the ID of the port the frame came from (i.e., 1). FWD-A does not contain any information about H 7 either, so the frame is sent out to ports 2 and 3 of the bridge. Bridge $B$ receives the frame at port 2, and the forwarding table of Bridge B (FWD-B) is empty, so it saves the information about H1 being reachable on port 2. The frame is also sent out to port 1. As a result, FWD-A and FWD-B are:

| FWD-A |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 1 |


| FWD-B |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 2 |

At $t=2$, H3 sends a frame to H 5 . The frame is received by Bridge B on port 1 . No entry in FWD-B about H3. H3 info saved together with port 1 ID. FWD-B has no information about H 5 , so the frame is sent out to port 2. Bridge A receives the frame at port 2. No entry in FWD-A about H3. H3 info saved together with port 2 ID. The frame is sent out to ports 1 and 3. As a result, FWD-A and FWD-B are:

| FWD-A |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 1 |
| $H 3$ | 2 |


| FWD-B |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 2 |
| $H 3$ | 1 |

At $t=3, \mathrm{H} 4$ sends a frame to H 3 . The frame is received by Bridge B on port 1 . No entry in FWD-B about H4. H4 info saved together with port 1 ID. FWD-B has information about H3. Knowing that H3 is reachable via port 1, the frame is not sent out to port 2. As a result, FWD-A and FWD-B are:

| FWD-A |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 1 |
| $H 3$ | 2 |


| FWD-B |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 2 |
| $H 3$ | 1 |
| $H 4$ | 1 |

At $t=4, \mathrm{H} 2$ sends a frame to H 4 . The frame is received by Bridge A on port 1. No entry in FWD-A about H2. H2 info saved together with port 1 ID. FWD-A has no information about H 4 , so the frame is sent out to ports 2 and 3. Bridge B receives the frame at port 2. No entry in FWD-B about H2. H2 info saved together with port 2 ID. FWD-B has information about H4, so the frame is sent out to port 1. As a result, FWD-A and FWD-B are:

| FWD-A |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 1 |
| $H 3$ | 2 |
| $H 2$ | 1 |


| FWD-B |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 2 |
| $H 3$ | 1 |
| $H 4$ | 1 |
| $H 2$ | 2 |

At $t=5, \mathrm{H} 8$ sends a frame to H1. The frame is received by Bridge A on port 3. No entry in FWD-A about H8. H8 info saved together with port 3 ID. FWD-A has information about H1, and the frame is sent out to port 1 and nowhere else. As a result, FWD-A and FWD-B are:

| FWD-A |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 1 |
| $H 3$ | 2 |
| $H 2$ | 1 |
| $H 8$ | 3 |


| FWD-B |  |
| :---: | :---: |
| Host address | Port ID |
| $H 1$ | 2 |
| $H 3$ | 1 |
| $H 4$ | 1 |
| $H 2$ | 2 |

(b) Transparent bridges (unlike hubs) have the property to separate the collision domains of the LANs connected to their ports.
We know that, for CSMA-CD to be effective, $t_{f} \geq 2 \cdot t_{\text {prop }}$. We must compute the worst-case propagation delay a frame from H 2 to H 3 goes through.
There are three collision domains to consider, L4, L2, and L3. In L4, the distance between H 2 and port 1 of Bridge A is 50 m . In L2, the distance a frame traverses from port 2 of Bridge A to port 2 of Bridge $B$ is 100 m . Finally, in L3, the distance between port 1 of Bridge B and H 3 is 50 m . The worst-case propagation delay (i.e., the max among the three) is 100 m . For this reason, we have:

$$
t_{\text {prop }}=100 / c=0.5 \mu \mathrm{~s}
$$

Knowing that the medium has a transmission rate equal to $R$, we can then derive the value of $n_{f}$ in bytes as:

$$
n_{f} \geq\left\lceil\left(2 \cdot t_{\text {prop }} \cdot R\right) / 8\right\rceil \Rightarrow n_{f} \geq 13 \text { bytes }
$$

In the case hubs are used, all hosts belong to the same collision domain. The distance between H2 and H3 is 200 m . This means that:

$$
t_{\text {prop }}=200 / c=1 \mu \mathrm{~s}
$$

Using the same procedure as above, we can derive that

$$
n_{f} \geq\left\lceil\left(2 \cdot t_{\text {prop }} \cdot R\right) / 8\right\rceil \Rightarrow n_{f} \geq 25 \text { bytes }
$$

Results are consistent with the notion that with longer propagation delays, we need to have a bigger value for the minimum frame length.
(c) Applying the Spanning Tree algorithm, we obtain the result in the figure below


While the spanning tree can be depicted as:

(d) If we would like to have a spanning tree solution where L5 is connected to the root bridge only, we need to enforce the following constraint

$$
C_{5} \geq C 2+C 3 \Rightarrow C_{5} \geq 6
$$

Reason: L1, L4, and L2 are connected directly to the root bridge. The cost of L5 does not impact them. On the other hand, we need to ensure that the L2 and L3
become more appealing options for L6 to connect to the root bridge compared to L5. The result is the following:

4. (a) What is the purpose of the modulator block in the Shannon communication model, and what are its main design criteria? (2p)
(b) Consider a vehicular traffic safety application based on broadcasting status messages (vehicle speed, heading, etc.). Which security goals (i.e., confidentiality, integrity, and authentication) are crucial in this application? Motivate. (3p)
(c) Explain the hidden terminal problem in Wi-Fi and how it can be mitigated using the RTS/CTS handshake. (3p)
(d) Explain the benefits and drawbacks of slotted versus unslotted Aloha. (2p)
(e) Consider a layer-n protocol. Which data unit has the most bits, a n-PDU or a n-SDU? Motivate. (2p)
(a) The modulator block converts digital signals to analog signals suitable for transmission over physical mediums (channels). The modulator is designed to maximize the data rate, minimize errors at the receiver and optimize the use of resources, i.e., power and bandwidth.
(b) Confidentiality is not an important goal in this application, as the shared information (vehicle speed, heading, etc.) is broadcasted to all nearby cars. So, the content is not confidential between particular users. Integrity is essential to ensure an intruder hasn't altered the messages sent. Authentication is also necessary to ensure that only authorized users can send these messages. The combination of these last two goals cancels the possibility of sending false messages that can trigger an unnecessary safety measure (e.g., braking) or false messages that result in non-taking safety measures in a dangerous situation.
(c) Let's consider a scenario where two terminals (A and C) would like to communicate wirelessly with a third terminal, B. A and C are both in the reach of B, but they are out of reach of each other. As a result, it might happen that A is not able to detect a transmission from C to B while sensing the channel (i.e., during its initial DIFS time). In this case, we say that C is hidden from A. With an RTS/CTS handshake, things happen differently. If A wants to transmit a frame, it must first send a Request to Send (RTS) frame to B. If B senses the channel is free, it will reply with a Clear to Send (CTS) frame back to A. Only at this point will A start data transmission. Otherwise (i.e., if B senses a busy channel or two concurrent RTS frames collide at B), B will not send out a CTS reply. If A does not receive a CTS frame, it will not start the data transmission.
(d) In Slotted Aloha, a station with a frame to transmit can only do so at the beginning of a time slot. As a result, the probability of collision decreases, and the throughput increases. Drawbacks: guard bands and synchronization among stations are required.
(e) A layer n-SDU is encapsulated into a layer n-PDU. As a result, a layer n-PDU has (in general) more bits than a layer n-SDU.

